

## An Interpretation of $\delta$ Scuti Stars

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Consistent absolute magnitudes are obtained for  $\delta$  Scuti stars using Strömgen intermediate-band colors and Crawford's photoelectric  $H\beta$  indices. A period-color-luminosity relation is found for these variables and the derived contours of constant period in the color-magnitude diagram are almost parallel to the zero-age main sequence. From the locations of 34 known variables and 168 possible nonvariables, two distinct instability regions are found in the H-R diagram; the short-period group lies very close to the zero-age main sequence, while the long-period group lies about 2 mag above the main sequence.

### I. INTRODUCTION

THE  $\delta$  Scuti stars are a group of variable stars which have the following properties: (i) spectral types ranging from A to early F, (ii) short periods, ranging from one to four hours, (iii) generally very small amplitudes in light and radial velocity (a few hundredths of a magnitude and 10 km/sec, respectively), (iv) variable light and radial-velocity curves in most cases, (v) a tendency to low space velocity, (vi) absolute visual magnitudes ranging from +0.5 to +2.5, (vii) population type approximately intermediate-age Population I. The original five members ( $\delta$  Scuti, DQ Cephei, CC Andromedae,  $\rho$  Puppis, and  $\delta$  Delphini) were first recognized by Eggen (1956a, 1956b) and by McNamara and Augason (1962), who studied their absolute luminosities and locations in the H-R diagram. The stars were found to be located above the main sequence. Analysis of their light curves has been carried out by Fitch (1960, 1967), Leung (1963), Wehlau and Leung (1964), Leung and Wehlau (1967), and Fulerton (1967). In general, the computed light curves derived from multiple periodicities represent the observations rather well. But the physical meaning of these components still has to be investigated. The relation between the amplitudes of light and radial velocity has been studied by Leung and Wehlau (1967). The effect of rotation on pulsations has been studied by Dickens (1968) and Breger (1969c). There are three extensive surveys in which a search for new members of  $\delta$  Scuti stars was made (Danziger and Dickens 1967; Millis 1967; Breger 1969a). At present there are forty-one variables with known periods which are classified as  $\delta$  Scuti stars. The effective temperatures and surface gravities of several members have been studied by Danziger and Dickens (1967), Millis (1967), and Bessell (1969). The solar motion and space velocity for a number of members has been investigated by Millis (1967), Bessell (1969), and Breger (1969b). Their motions are in accord with those of normal Population I stars.

Since most of the  $\delta$  Scuti stars are bright stars of spectral types A and F, *uvby* colors and  $H\beta$  indices

can be found for them in the catalogues of Strömgen and Perry (1965) and Crawford *et al.* (1966). These provide us with *consistent* intrinsic colors and absolute luminosities for  $\delta$  Scuti stars. The relative locations between the variables and the possible nonvariables [from the lists of Danziger and Dickens (1967), Millis (1967), and Breger (1969b)] in an H-R diagram might give us some insight as to the region of instability and evolutionary stages of these stars. It will also be of interest to investigate the possibility of a period-color-luminosity relation for  $\delta$  Scuti stars.

### II. THE COLOR INDICES, AND ABSOLUTE VISUAL LUMINOSITIES OF $\delta$ SCUTI STARS

The number of  $\delta$  Scuti stars has grown from five to over forty during the last few years. The majority of the new members were discovered from the extensive surveys by Danziger and Dickens (1967), Millis (1967), and Breger (1969a). The rest were discovered by Landolt (1966), Jones (1966), Jones and Lagerwey (1966), Chen (1966), Wehlau *et al.* (1966), Dickens (1967), Eggen (1968), Breger and Sanwal (1968), and Cousins *et al.* (1969). Data for all the known members have been collected in Table I. The values of  $(b-y)$ ,  $m_1$ ,  $c_1$  are taken from the Strömgen and Perry catalogue (1965), Cameron's catalogue (1966), and Crawford and Barnes (1969). The  $\beta$  indices are taken from the catalogue of Crawford *et al.* (1966) and Crawford and Barnes (1969). The quantities  $[m_1]$  and  $[c_1]$  are reddening-independent colors calculated according to the standard formula given by Strömgen (1966) (assuming a standard reddening law).  $\Delta[c_1]$  is the difference between  $[c_1]$  for the star in question and the value of  $[c_1]$  for the zero-age main sequence which corresponds to the same  $\beta$  value. The zero-age  $[c_1]-\beta$  sequence is derived from Crawford and Barnes (1969, Table III). The intrinsic color  $(b-y)_0$  and absolute visual magnitude  $M_V$  were calculated from the equations given by Strömgen (1966) for his "late group" of stars, here restated as the following:

$$(b-y)_0 = 2^m 946 - \beta - 0.15 \Delta[c_1], \quad (1)$$

$$M_V = 1^m 6 + 7.5(2^m 946 - \beta) - 11.0 \Delta[c_1]. \quad (2)$$

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TABLE I. Known  $\delta$  Scuti stars.

	Name	Sp.	Period	(b−y)	m <sub>1</sub>	c <sub>1</sub>	β	[m <sub>1</sub> ]	[c <sub>1</sub> ]	Δ[c <sub>1</sub> ]	(b−y) <sub>0</sub>	M <sub>V</sub>	
HR	21	β Cas	F2 IV	0.104	0.216	0.177	0.785	2.709	0.215	0.742	0.217	0.204	0.99
HR	114	28 And	Am	0.069	0.169	0.165	0.869	2.755	0.195	0.835	0.175	0.183	1.24
HR	242	ρ Phe	dF2	0.11:									
HR	431		F0 IV	0.090									
HR	432	97 Psc	A4 III	0.16	0.090	0.166	1.093	2.817	0.182	1.075	0.255	0.108	−0.10
HR	515		A7n	0.16	0.158	0.173	0.979	2.777	0.201	0.947	0.277	0.153	0.50
HR	729	26 Ari	A4n	0.06	0.155	0.185	0.839	2.777	0.213	0.808	0.088	0.174	2.03
HR	812	38 Ari	A7 IV	0.037	0.136	0.185	0.841	2.804	0.209	0.814	0.045	0.135	2.17
HR	1170		F0	0.091	0.179	0.154	0.826	2.743	0.186	0.790	0.166	0.196	1.43
HR	1223		A5	0.046	0.124	0.190	0.818	2.801	0.212	0.793	0.009	0.162	2.72
HR	1287	44 Tau	dF3	0.132	0.215	0.175	0.752	2.708	0.214	0.709	0.189	0.211	1.31
HR	1351	57 Tau	F0	0.062	0.172	0.197	0.770	2.767	0.228	0.736	0.036	0.174	2.55
HR	1547	97 Tau	dF5	0.042	0.132	0.195	0.900	2.813	0.219	0.874	0.061	0.124	1.93
HR	1611	64 Eri	F0 IV										
HR	1653		F2 IV	0.135									
HR	1706	14 Aur	A9	0.12	0.132	0.180	0.998	2.799	0.203	0.972	0.184	0.120	0.66
HR	2107	1 Mon	F2 II	0.137	0.153	0.197	0.888	2.740	0.224	0.857	0.241	0.170	0.49
HR	2539	59 Aur	A7n		0.240	0.186	0.764	2.702	0.229	0.716	0.220	0.211	1.00
HR	2707	21 Mon	A8n	0.11	0.184	0.187	0.878	2.740	0.220	0.841	0.225	0.172	0.67
HR	3185	ρ Pup	F6 II–III	0.141	0.246	0.218	0.750	2.695	0.262	0.701	0.225	0.217	1.00
HR	3265		A5	0.08	0.196	0.230	0.786	2.753	0.265	0.747	0.091	0.179	2.05
HR	3888	ν UMa	F2 IV	0.133	0.196	0.160	0.830	2.738	0.197	0.791	0.183	0.181	1.15
HR	4684		A7	0.055	0.103	0.196	0.938	2.835	0.214	0.917	0.055	0.103	1.83
HR	4715	4 CVn	F0	0.17	0.226	0.178	0.833	2.707	0.219	0.788	0.275	0.198	0.40
HR	5005		F0	0.14	0.200	0.166	0.824	2.720	0.202	0.784	0.224	0.192	0.83
HR	5017	20 CVn	F0 II–IIIp	0.122	0.180	0.231	0.913	2.778	0.263	0.877	0.145	0.146	1.27
HR	5329	κ Boo	A7 IV	0.069									
HR	5788/9	δ Ser	F0 IV	0.13:									
HR	5960		F0 IV	0.062	0.178	0.188	0.776	2.761	0.220	0.740	0.060	0.176	2.33
HR	6391	63 Her	A3	0.077	0.122	0.200	0.901	2.798	0.222	0.877	0.101	0.151	1.74
HR	6581	ο Ser	A2 IV										
HR	7020	δ Sct	F3 III–IV	0.194	0.192	0.206	0.824	2.741	0.240	0.785	0.169	0.180	1.17
HR	7222		F2	0.096	0.206	0.168	0.168	2.749	0.205	0.851	0.211	0.183	0.87
HR	7331	28 Aql	F0	0.157	0.164	0.184	0.990	2.796	0.214	0.957	0.183	0.140	0.85
HR	7340	ρ Sgr	F0 IV										
HR	7501		F0	0.082	0.222	0.187	0.768	2.728	0.229	0.724	0.144	0.214	1.79
HR	7563		F0 III	0.100									
HR	7928	δ Del	A7 III	0.135	0.190	0.163	0.854	2.738	0.197	0.816	0.208	0.177	0.87
HR	8006		F0	0.06	0.171	0.184	0.810	2.760	0.215	0.776	0.096	0.190	2.07
HR	8157		F2		0.332	0.151	0.742	2.665	0.211	0.676	0.290	0.237	0.50
HR	8494	ε Cep	F0 IV	0.042	0.167	0.194	0.790	2.757	0.224	0.757	0.091	0.175	2.02
HR	8584		A5	0.056	0.118	0.193	0.959	2.821	0.214	0.935	0.107	0.127	1.50
		CC And	F3 IV–V	0.125									
		DQ Cep	F1 IV	0.079	0.179	0.228	0.844	2.739	0.260	0.808	0.196	0.178	1.00
HD	24550		sg A8n	0.076									
HD	116994		A5	0.102									

Notice that the coefficient for the  $\Delta[c_1]$  term is relatively large in Eq. (2). Thus  $M_V$  is sensitive to  $\Delta[c_1]$ . Since conventional colors are in Johnson's  $UBV$  system,  $(b-y)_0$  has been transformed into  $(B-V)_0$  by means of the relation given by Crawford and Perry (1966). All the calculated quantities described are also listed in Table I.

We would like to investigate how Strömgren's calibration [Eq. (2)] applies to variable stars of  $\delta$  Scuti type. Fortunately, there are 13 members for which independent estimates of absolute visual magnitudes are available from sources not employing multicolor photometry: cluster parallax, companion of a binary, trigonometric parallax ( $\pi \geq 0''.025$ ), moving-group parallax. The moving-group parallaxes are to be taken as having very low weight (Breger 1968) while the cluster parallaxes and companion in a binary system are to be

taken as having higher weight. A comparison between the value of  $M_V$  derived from Strömgren's photometry and from other independent sources is listed in Table II. The values are in good agreement except for 20 CVn (from possible membership in the Hyades moving-group), and  $\rho$  Pup (from three highly discordant trigonometric parallaxes). The values in Table II are plotted in Fig. 1. Notice that the most reliable values of  $M_V$ , which are derived from cluster membership and from a companion in a binary system, lie very close to the 45-deg line drawn through the origin, while the others fall on either side of it. Since there is no serious systematic deviation from the 45-deg line, there is no need to apply a zero-point correction to Strömgren's photometric  $M_V$ . The rms residual is about 0.3 mag which is similar to the order of accuracy expected from Strömgren's photometry, p.e. in  $M_V$  being of the order

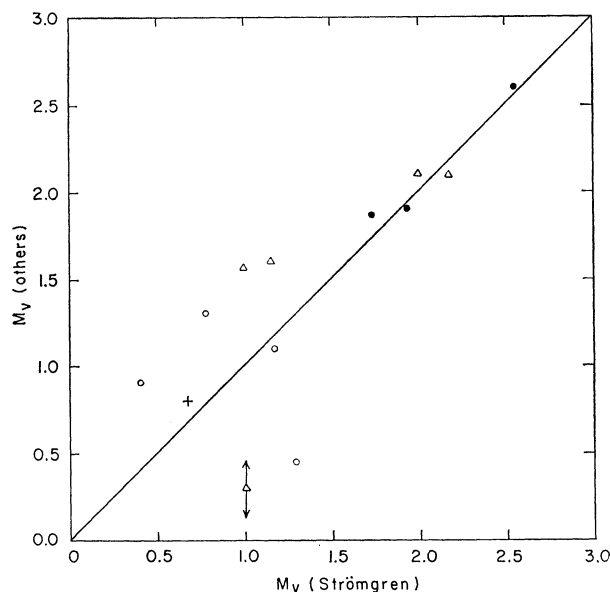


FIG. 1.  $M_V$  from Strömgren photometry vs  $M_V$  from cluster membership (filled circles); companion of a binary system (cross), membership in a moving group (circles), trigonometric parallax (triangles).

of  $\pm 0^m15$  (Strömgren 1966). Thus, we conclude that we may apply Strömgren's photometric  $M_V$  to  $\delta$  Scuti stars with confidence. In order to investigate accurately the location of the  $\delta$  Scuti stars in the H-R diagram, we should employ only those observations which are homogeneous. Therefore in this paper we use only those colors and values of  $M_V$  which are derived from Strömgren's four colors and Crawford's  $H\beta$  photometry.

### III. LARGE $[m_1]$ STARS

The majority of the variables are clustered in the area around 0.80 and 0.21 of  $[c_1]$  and  $[m_1]$ , respectively. Four stars ( $\rho$  Pup, HR 3265, 20 CVn, and DQ Cep) have very large  $[m_1]$  values ( $\geq 0^m26$ ). The intermediate  $[m_1]$  star ( $\sim 0^m24$ ) is  $\delta$  Sct. The  $[m_1]$  index of Strömgren corresponds very well with the Fe/H abundance. This suggests that the stars mentioned above may be related to the metallic-line stars. Curve-of-growth analyses have been applied to three members ( $\rho$  Pup,  $\delta$  Sct, and  $\delta$  Del) by Bessell (1969). He has found that the first two stars have an overabundance of iron with respect to the normal F-type star  $\eta$  Lep, suggesting that these stars resemble metallic-line stars. In Table III the  $[m_1]$  indices and the iron abundances of the stars studied by Bessell are listed. We notice that the  $[m_1]$  index is well correlated with the iron abundance for the  $\delta$  Scuti stars studied. A large  $[m_1]$  indicates strong-line stars. This may result from overabundances, microturbulence, or something else.

Morgan (Abt 1963) suggested that  $\delta$  Del and  $\beta$  Cas

TABLE II. Comparison of  $M_V$  of  $\delta$  Scuti stars by Strömgren's photometry and by other sources.

Stars	$M_V$ (in mag)	
	Other sources	Strömgren's
HR 21, $\beta$ Cas	1.56 <sup>a</sup>	0.99
HR 812, 38 Ari	2.1 <sup>a</sup>	2.17
HR 1351, 57 Tau	2.6 <sup>b</sup>	2.55
HR 1547, 97 Tau	1.9 <sup>b</sup>	1.93
HR 1706	0.8 <sup>c</sup>	0.66
HR 3185, $\rho$ Pup	0.3 <sup>b</sup>	1.00
HR 3888, 29 $\nu$ UMa	1.6 <sup>a</sup>	1.15
HR 4684	1.89 <sup>d</sup>	1.72
HR 4715, 4 CVn	0.9 <sup>e</sup>	0.40
HR 5017, 20 CVn	0.45 <sup>f</sup>	1.27
HR 7020, $\delta$ Sct	1.1 <sup>f</sup>	1.17
HR 7928, $\delta$ Del	1.3 <sup>g</sup>	0.89
HR 8494, $\epsilon$ Cep	2.1 <sup>a</sup>	2.02

<sup>a</sup> From trigonometric parallax ( $\pi \geq 0''.025$ ).

<sup>b</sup> Danziger and Dickens (1967) from membership in Hyades cluster.

<sup>c</sup> Danziger and Dickens (1967) from companion of a binary.

<sup>d</sup> Breger and Sanwal (1968) from membership in Coma Cluster.

<sup>e</sup> Danziger and Dickens (1967) from possible membership in Hyades moving-group.

<sup>f</sup> Danziger and Dickens (1967) from possible membership in Sirius moving-group.

<sup>g</sup> From the mean of three published parallaxes,  $0''.027$  McCormick,  $0''.019$  Yale, and  $0''.049$  Cape.

might be examples of transition cases of metallic-line stars. The  $[m_1]$  values of these two stars (Table I) are normal.  $\delta$  Del was discovered by Preston (quoted by Danziger and Dickens 1967) to be a spectroscopic binary with a period of 40.5 d and identical components.

Strömgren (1963, 1966) noticed that there should be a small correction to the photometric  $M_V$  for stars with large  $[m_1]$ . The star 20 CVn has the largest discrepancy in  $M_V$  (Table II) and has a large  $[m_1]$  value of 0.263. Since the correction to the photometric  $M_V$  due to large  $[m_1]$  for  $\delta$  Scuti stars is not now well known, we assume that the values of  $M_V$  for the stars with large  $[m_1]$  are of lower weight than for the others.

### IV. P-C-L AND P-L RELATION

A period-color relation for  $\delta$  Scuti stars was suggested by Millis (1967, 1968). He found that the  $(B-V)$  color tends to be a linear function of the period. The intrinsic colors  $(b-y)_0$  and the periods of the stars in Table I indicate only a very weak correlation. However, the color spread for  $\delta$  Scuti stars in  $(b-y)_0$  is small ( $\sim 0^m1$ ).

Dickens (1968) suggested that there may be a weak period-luminosity relation for  $\delta$  Scuti stars. Leung (1969) derived an observational P-C-L relation for

TABLE III.  $[m_1]$  index and iron abundance of some  $\delta$  Scuti stars.

	$\rho$ Pup	$\delta$ Sct	$\delta$ Del	$\alpha$ CMi	$\eta$ Lep
Sp.	F6 III-IV	F3 III-IV	A7 III, F4 IV	F5 IV	F0 V
$[m_1]$	0.262	0.240	0.197	0.216	0.202
[Fe I]*	0.36	0.39	-0.11	-0.04	0.0
[Fe II]*	0.36	0.39	-0.11	-0.04	0.0

\* With respect to  $\eta$  Lep (Bessell 1969).

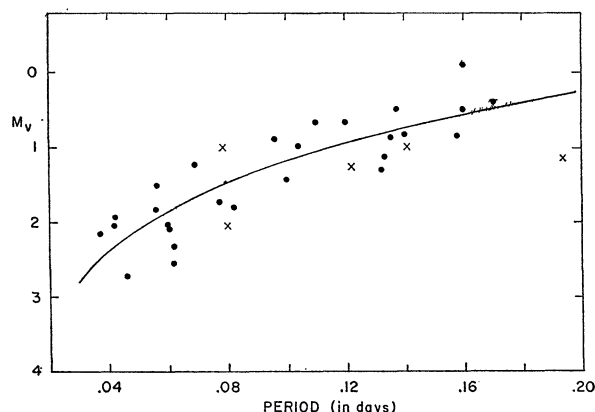


FIG. 2. Period vs absolute visual magnitude: normal  $[m_1]$  stars (filled circles), large  $[m_1]$  stars (crosses).

these variables (20 stars). Independently, Breger (1969c) derived a semitheoretical P-C-L relation combining: (i) Fernie's (1965) period-radius-mass relation, and (ii) a color-effective-temperature relation.

In Table I, there are 32 stars which have known periods and  $ubvy$  and  $\beta$  photometry. Among them, five stars have relatively large  $[m_1]$  indices. As discussed earlier, the absolute visual magnitudes derived from the Strömgren photometry for these stars are less certain. A least-squares solution for the remaining 27 stars leads to the following P-C-L relation:

$$M_V = -3.01 - 3.23 \log P + 5.6(b-y)_0 \quad (P \text{ in days}). \quad (3)$$

The probable error in  $M_V$  from Eq. (3) is  $\pm 0^m22$  which is similar to the order of accuracy of Strömgren's calibration of  $\pm 0^m15$  (Strömgren 1966). Since Eq. (3) gives a small probable error in  $M_V$ , we conclude that the P-C-L relation found is quite satisfactory. Because the color  $(b-y)_0$  spread is small, a P-L relation may also exist for these stars. A least-squares solution from the observational data for the same stars leads to the following P-L relation:

$$M_V = -1.88 - 3.06 \log P \quad (P \text{ in days}). \quad (4)$$

The probable error in  $M_V$  from Eq. (4) is  $\pm 0^m25$  which is slightly larger than the similar value from Eq. (3). Figure 2 is a plot of absolute visual magnitude  $M_V$  against period  $P$ . The P-L relation of Eq. (4) is shown as a solid line in the same diagram.

The coefficient of the  $\log P$  term in the P-C-L relation [Eq. (3)] is  $-3.2$ , which is very close to the theoretical value of

$$\begin{aligned} -3.3[\text{i.e., } M_{\text{bol}} - M_{\text{bol}\odot} \\ = -3.3 \log P - 10 \log(T_e/T_{e\odot}) \\ + 3.3 \log Q - (10/6) \log(\mathfrak{M}/\mathfrak{M}_\odot)] \end{aligned}$$

and different from the value  $-2.5$  adopted by Breger (1969b). The coefficient for the color term is  $5.6$  which

is very similar to the value of  $5.5$  found by Breger (1969b) in quite a different manner.

If the large  $[m_1]$  stars are included, the P-C-L and P-L relations become

$$M_V = -2.58 - 2.94 \log P + 5.1(b-y)_0 \quad (P \text{ in days}) \quad (5)$$

and

$$M_V = -1.53 - 2.76 \log P \quad (P \text{ in days}). \quad (6)$$

The probable errors in  $M_V$  from Eqs. (5) and (6) are  $\pm 0^m25$  and  $\pm 0^m26$ , respectively.

#### V. THE H-R DIAGRAM OF THE VARIABLES AND THE POSSIBLE NONVARIABLE A AND F STARS

An H-R diagram for all known variables (34 stars) with Strömgren and Crawford photometry is shown in Fig. 3. The stars with undetermined periods are represented by open circles. The number attached to the symbol is the period of the star expressed in hundredths of a day. The zero-age main sequence of Strömgren (1963) is denoted as a solid line in the same figure. The stars seem to be polarized into two groups, which are indicated by broken boxes. The upper box consists of variables with longer periods than those in the lower box. The locations of these two boxes are found to coincide with the linear extension of the Cepheid instability strip as suggested earlier by Danziger and

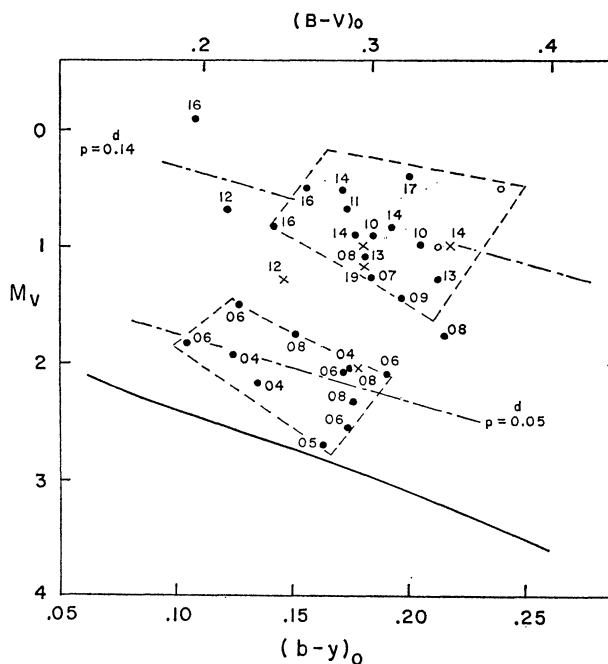


FIG. 3. H-R diagram of  $\delta$  Scuti stars: normal  $[m_1]$  stars (filled circles), large  $[m_1]$  star (crosses), stars with unknown periods (open circles). Number attached to the symbol indicates the period in hundredths of a day. Also indicated are the instability regions (dotted-line boxes), period contours (dotted lines), and zero-age main sequence (solid line).

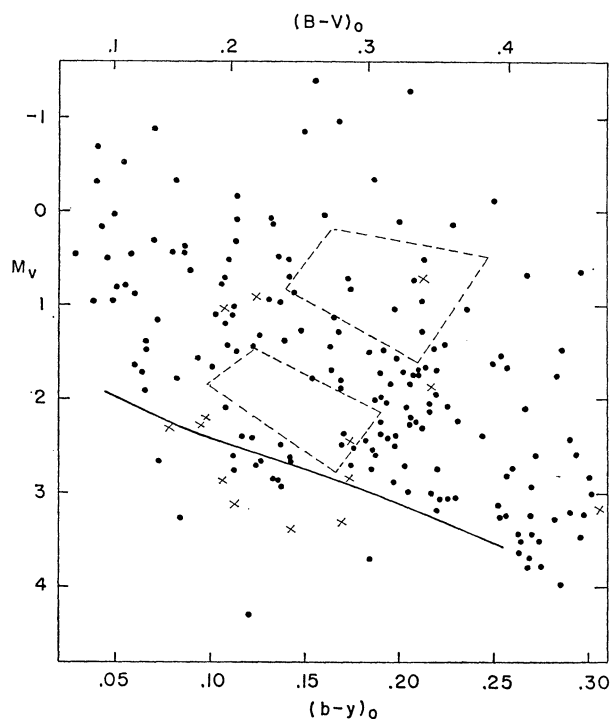


FIG. 4. H-R diagram of possible nonvariable stars (see text), with instability regions (dotted-line boxes) and zero-age main sequence (solid lines) indicated.

Dickens (1967). There are four stars, HR 432 (97 Psc), HR 1706 (14 Aur), HR 5017 (20 CVn), and HR 7501 [in order of increasing  $(b-y)_0$ ], which fall outside these two instability regions. HR 432 has the spectrum of an early giant. HR 1706 is the bright component of a wide visual binary (14".6 apart) and itself a spectroscopic binary (Harper 1916). It is an unusual variable in many aspects (Chevalier *et al.* 1968; Hudson *et al.* 1970). HR 5017 has a peculiar spectrum of a giant star and an extremely high  $[m_1]$  index of 0.263. According to Breger (1969a), "HR 7501 has a very small variable amplitude of about  $0^m.01$  which at times approaches the limit of detectability."

From Eq. (3) we can calculate the contours of constant period in the H-R diagram. The period contours for  $0^d.05$  (1.2 h) and  $0^d.14$  (3.3 h) are indicated as straight broken lines in Fig. 3. Notice that the period contours are almost parallel to the ZAMS and represent the observations well. Christy (1966) has given an expression for the fundamental period of a variable inside the instability strip,

$$P_0 \approx 0.022 (g_0/g)^{7/8} (\mathcal{M}/\mathcal{M}_\odot)^{1/8},$$

showing that the period is determined almost entirely by the value of the effective gravity. Comparing the constant-period contours in Fig. 3 with the constant-gravity lines of Bessell (1969, Fig. 10), one finds qualitative agreement. Quantitatively, periods of 3.05 d

for  $\log g \approx 4.1$  and 0.14 d for  $\log g \approx 3.6$  are also in good agreement with theoretical prediction. [I am indebted to the referee for pointing out this agreement.]

The three major surveys for  $\delta$  Scuti stars of Danziger and Dickens (1967), of Millis (1967), and of Breger (1969a, 1969b) suggested a large number of possible nonvariable stars with spectral types of A and F, and luminosity classes of IV to III. Out of this large number of stars, 168 have published *ubvy* and  $\beta$  measurements. Most of the  $\delta$  Scuti stars have very small amplitudes in light variation and variable light curves. If it happened that the survey was carried out during the phase when the variable had an extremely small light amplitude or even constant light, we would not be able to recognize it as a variable star. Under extensive observation we may find that some of the nonvariable stars are actually variable. The possible nonvariables are collected in this paper. [Note that Ap stars, stars with only one observation and two known binaries (Tr 104 and 109, Crawford and Barnes 1969) are excluded.] The  $\beta$ ,  $(b-y)$ ,  $m_1$ , and  $c_1$  indexes are taken from the catalogues of Crawford *et al.* (1966), Strömgren and Perry (1965), and Cameron (1966).  $M_V$  and  $(b-y)_0$  are calculated by the method described earlier. The results are plotted in Fig. 4. The solid line in the same diagram denotes the ZAMS. The Strömgren calibrations [Eqs. (1) and (2)] become less accurate for stars which are bluer than 0.05 and redder than 0.25 in  $(b-y)_0$ . The broken boxes are the locations of the short- and long-period groups of  $\delta$  Scuti stars. It is clear from the H-R diagram that nearly all the possible nonvariables are located *outside* the boxes. There are a few possible nonvariables that fall into the two instability regions or boxes. The star counts for variables and possible nonvariables are summarized in Table IV. It is difficult to apply the usual statistical tests because the selection effect inherent in the data (the three major surveys all had different selection criteria and the rest of the stars were discovered in an accidental manner) and we do not have a good estimate of the true or expected distribution of this type of star (the evolutionary stages of these stars are not known, see Sec. VI). Unless there is some systematic effect present, we would probably not expect such a

TABLE IV. Statistic of the variables and non-variables in the H-R diagram.

	Lower box	Upper box	Between boxes	Outside boxes
Variable	12 (11)	18 (15)	1 (0)	4 (3)
Nonvariable	5 (4)	9 (8)	21 (21)	154 (142)
Total	17	27	22	158
Ratio = $\frac{\text{variable}}{\text{nonvariable}}$	$\sim 2.5$	$\sim 2$	$\sim 0$	$\sim 0$

Note: The bracketed numbers are the star counts excluding the large  $[m_1]$  and Am stars.

TABLE V. Possible nonvariables inside instability regions.

Name	Sp.	$b-y$	$m_1$	$c_1$	$\beta$	$[m_1]$	$[c_1]$	$(b-y)_0$	$M_V$	Time obs. (in hours)	Remarks
Lower region											
HR 238	gF6	0.166	0.216	0.780	2.770:	0.245	0.747	0.170	2.49	0.7	Large $[m_1]$ , sp. of F6
HR 457	A5	0.146	0.193	0.847	2.777	0.219	0.818	0.154	1.79	3.0	Lies on the boundary
HR 687	dF2	0.186	0.158	0.768	2.764	0.191	0.731	0.176	2.51	0.7	Lies on the boundary
HR 1331	dA8	0.175	0.185	0.787	2.767	0.216	0.852	0.171	2.37	3.5	Has variation of $\pm 0^m.01$
HR 7774	Am	0.196	0.206	0.775	2.764	0.241	0.736	0.175	2.46	2.6	Am star
Upper region											
HR 214	A5	0.165	0.178	0.929	2.773	0.208	0.896	0.145	0.85	3.9	Lies on the boundary
HR 840	F2 III	0.220	0.174	0.761	2.701	0.214	0.717	0.211	0.96	2.5	
HR 3885	F0	0.173	0.181	0.871	2.740	0.212	0.885	0.173	0.71	3.4	
HR 4480	dF2	0.237	0.186	0.761	2.704	0.228	0.714	0.213	1.28	2.5	
HR 6560	F2	0.373	0.220	0.805	2.695	0.287	0.730	0.213	0.71	4.6	Extremely large $[m_1]$
HR 6604	dF4	0.273	0.168	0.798	2.701	0.217	0.743	0.208	0.74	7.2	
HR 7916	F0	0.223	0.186	0.806	2.704	0.226	0.769	0.203	0.52	1.1	
HR 8198	F0	0.208	0.158	0.795	2.716	0.195	0.753	0.197	1.03	11.0	
HR 8267	F0 IV	0.203	0.170	0.890	2.734	0.207	0.849	0.174	0.83	8.0	

large deviation as shown in Table IV. We conclude that the existing two instability regions indicated in Figs. 3 and 4 are significant.

The stars which fall in the two boxes in Fig. 4 deserve further attention. They may be truly nonvariable, variable, or binary (in the last case, the present location in the H-R diagram may be wrong). These stars are listed in Table V.

The star HD 24550 has no Strömgren colors, but it is believed to be the brighter component of the triple system ADS 2849. Dickens (1967) derived the following quantities for this star:  $M_V=0.58$ ,  $(B-V)_0=0.34$ , and  $P=0.0757690$  d. We notice that this star does not fit the P-L relation (Fig. 2) very well, but it does fall within the upper instability region (Fig. 3). [I am indebted to Dr. N. Baker for drawing my attention to this star.]

It is suspected that the reason why the previous surveys (Danziger and Dickens 1967; Millis 1967; and Breger 1969b) did not find a separation between the variables and the possible nonvariables may be due to the fact that the dimensions of the instability regions are rather small. One may have to use a homogeneous system of  $M_V$  and  $(b-y)_0$  which is derived from better calibration like Eqs. (1) and (2). The probable errors in  $(b-y)$ ,  $m_1$ ,  $C_1$  (for two observations, Strömgren and Perry 1965), and  $\beta$  (for four observations, Crawford *et al.* 1966) are  $\pm 0.004$ ,  $\pm 0.005$ ,  $\pm 0.006$ , and  $\pm 0.006$  of a magnitude, respectively. These probable errors lead to an internal probable error in  $M_V$  [from Eq. (2)] of the order of  $\pm 0.2$  of a magnitude which is almost half the size of the lower box and of the region between

boxes. Thus, we can see that the inferior observations or observational quantities derived from inhomogeneous sources may cause difficulty in resolving the variables and the possible nonvariables in an H-R diagram.

## VI. MASS AND EVOLUTIONARY STAGE

There are three short-period  $\delta$  Scuti stars (HR 1351, HR 1547, HR 4684) which are members of clusters (Table II). These  $\delta$  Scuti stars have ages of the order of  $4 \times 10^8$  yr and masses of about  $1.7 M_\odot$ . They are located in the lower box of Fig. 3. It is quite likely that all the short-period  $\delta$  Scuti stars located in the lower box have masses around  $1.7 M_\odot$  and are in the hydrogen-burning phase of evolution.

There are three long-period  $\delta$  Scuti stars ( $\rho$  Pup,  $\delta$  Sct,  $\delta$  Del) which are located in the upper box and have reliable determinations of surface gravity and effective temperature (Bessell 1969). In Sec. II we have seen that the Strömgren and Crawford photometry is a reliable method to derive the absolute magnitudes of  $\delta$  Scuti stars. Thus we can determine the masses of the variables if the effective temperatures are surface gravities are known. By this method Bessell (1969) derived the masses of three  $\delta$  Scuti stars ( $\rho$  Pup,  $\delta$  Sct,  $\delta$  Del) in the long-period box to be around  $2 M_\odot$ . It is quite likely that the masses of all the variables in the long-period box are of the order of  $2 M_\odot$ . These stars can be either in an immediately post-main-sequence (PMS) core-contraction stage (evolving from left to right), or in a post-helium-flash (PHF) helium-burning stage (horizontal-branch stage).

The arguments for or against these two cases are outlined in the following paragraphs.

The observed masses and the location of the upper box agree very well with theoretical predictions for stars of around  $2\mathcal{M}_{\odot}$  during their PMS phase of evolution from left to right in the H-R diagram (Iben 1967a, b). However, this phase of evolution is extremely rapid (Iben 1967b, stages 7 to 8), and it is rather unlikely that we would discover so many stars during this phase of evolution from a more or less random sample.

According to the study of Iben (1967b), the smallest mass for *no helium flash* in Population I stars lies between 2.25 and  $3\mathcal{M}_{\odot}$ . Therefore it is quite likely that Population I stars of about  $2\mathcal{M}_{\odot}$  would undergo the helium flash and burn helium in a horizontal-branch phase. During this phase the evolutionary lifetime is much longer than during the rapid PMS phase. Unfortunately, due to the difficulty in handling the problem of the helium flash there are no evolutionary tracks available for  $\sim 2\mathcal{M}_{\odot}$  stars which have gone past this stage of evolution. From the theoretical information on Population II stars gleaned by Faulkner (1966a, b) and Iben and Faulkner (1968), we know that the locus of the helium-burning phase (horizontal-branch phase) is very sensitive to the chemical composition (affecting the effective temperature) and the core mass (affecting the luminosity). Therefore it may be possible that all or some of the stars in the upper box are post-helium-flash horizontal-branch stars.

## VII. CONCLUSION

Strömgren and Crawford photometry is a good method for obtaining consistent absolute visual magnitudes for  $\delta$  Scuti stars. Period-color-luminosity and period-luminosity relations exist for this group of variables. There are two distinct instability regions in the H-R diagram: a region of short-period stars lying close to the main sequence and a region of long-period stars lying well above the main sequence. The short-period stars have masses of about  $1.7\mathcal{M}_{\odot}$  and are probably in the hydrogen-burning phase of evolution. The long-period stars have masses of the order of  $2\mathcal{M}_{\odot}$ . They may be in the PMS core-contraction phase or the PHF helium-burning phase of evolution. There seems to be a "zone of avoidance" between the two regions of variables in the H-R diagram. The interpretation of this zone is very difficult. It is suggested here that there might be interference of different pulsation modes in this zone on the H-R diagram, causing an irregular, low-amplitude pulsation not readily detected; or perhaps the excitation mechanisms for the two groups are different.

In order to have a better understanding of  $\delta$  Scuti stars we would need to have detailed post-helium-flash model tracks for  $\sim 2\mathcal{M}_{\odot}$  stars. Detailed pulsating

models should also be extended to the study of  $\delta$  Scuti stars. Since most major surveys are carried out in the northern hemisphere, an extensive survey for  $\delta$  Scuti stars in the southern hemisphere will no doubt improve the statistics of variables and possible nonvariables. One such homogeneous survey could be a good statistical test for the existence of the instability regions. Extremely accurate Strömgren and Crawford photometry for the additional stars (for reasons outlined in Sec. V) are required to improve (or confirm) the locations of the two instability regions.

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